澳 門 大 學

澳 門 旅 遊 大 學

Macao University of

( )

Joint Admission Examination for Macao Four Higher Education Institutions
(Languages and Mathematics)

2024
2024 Examination Paper and Suggested Answer

Mathematics Standard Paper

<u>第一部份</u> 選擇題。請選出每題之

1. $A=\{\ :\ ^2-3\ -4\le 0\}$ $B=\{\ :3\ +a\ge 0\}$ $A\cap B=\{\ :2\le\ \le 4\}$

   $a=(\quad)$

   A.-12 B. -6 C. -3 D. 6 E. 12

2. $f(\ )=f(\ +1)+1$ $f(0)=16$ $f(15)$ $(\quad)$

   A. 0 B. 1 C. 15 D. 16 E. 17

3. $4\ +5\ =\ (\ +1)-(\ -1)(\ -1)$ 4 $(\quad)$

   A. 8 B. 4 C. 2

   D. 4 E. 8

4. $(\sqrt{\ }-2)^5(2\ -1)^4$ $(\quad)$

   A. -182 B. -178 C. 176 D. 178 E. 184

5. $P(2,3)$ - M P M

   M $(\quad)$

   A. $^2+\ ^2-13=0$ B. $^2+\ ^2+4\ -6\ =0$ C. $^2+\ ^2+4\ +6\ =0$

   D. $^2+\ ^2-4\ -6\ =0$ E. $^2+\ ^2-4\ -6\ +13=0$

6. $\dfrac{3\log\frac{1}{2}+\log 16}{\log 4+\log 5-1}=(\quad)$

   A. 1 B. -1 C. 2 D. -2 E. 4

7. 2 5 9 7 10 288 20

   $(\quad)$

   A. 32768 B. 65536 C. 131072 D. 262144 E. 524288

8. $\{\ ,\ \ -3,2\ +5,4\ -4,5\ +10\}$ 6.8 $(\quad)$

   A. 4 B. 5 C. 15 D. 0 E. -1

9. $\sqrt{1+\left(\dfrac{m^4-1}{2m^2}\right)^2}=(\quad)$

A. $\dfrac{m^4+2m+1}{2m^2}$ B. $\dfrac{m^4-1}{2m^2}$ C. $\dfrac{m^2}{2}+\dfrac{1}{2m^2}$ D. $\dfrac{\sqrt{m^2+1}}{2}$ E.

10. $\triangle ABC$ $|AB|=8$ $|AC|=7$ $\sin C=\dfrac{4\sqrt{3}}{7}$ $|BC|=(\quad)$

A. 6 B. 12 C. 2 D. 3 E. 5

11. $(-2,0)$ $(6,0)$ $(0,4)$ $(m,\ )$

$(\quad)$

A. $\dfrac{8}{3}$ B. $\dfrac{16}{3}$ C. 4 D. 8 E. 16

12. $(\quad)$ $f(\ )=5\cos(\ +\dfrac{\pi}{3})$

A. $\left(0,\dfrac{\pi}{2}\right)$ B. $\left(\dfrac{\pi}{2},\pi\right)$ C. $\left(\pi,\dfrac{3\pi}{2}\right)$

D. $\left(\dfrac{3\pi}{2},2\pi\right)$ E. $\left(\dfrac{\pi}{3},\dfrac{5\pi}{6}\right)$

13. $\theta\in[0,\pi)$ $1+\sin\theta-2\cos^2\theta=0$ $\theta=(\quad)$

A. $\dfrac{\pi}{6}$ $\dfrac{5\pi}{6}$ B. $\dfrac{\pi}{3}$ C. $\dfrac{\pi}{6}$ $\dfrac{\pi}{3}$ D. $\dfrac{\pi}{6}$ $\dfrac{\pi}{2}$ E. $\dfrac{\pi}{3}$ $\dfrac{\pi}{2}$

14. ${}^2-3\ +1=0$ ${}^4+\dfrac{1}{{}^4}=(\quad)$

A. 2 B. 47 C. 49 D. 79 E. 81

15. $f(\ )$ $\mathbb{R}$ $(-\infty,0)$ $(\quad)$

A. $f(2^{-\frac{7}{3}})>f(3^{-\frac{2}{7}})>f(\log_3\frac{2}{7})$ B. $f(3^{-\frac{2}{7}})>f(\log_3\frac{2}{7})>f(2^{-\frac{7}{3}})$

C. $f(\log_3\frac{2}{7})>f(2^{-\frac{7}{3}})>f(3^{-\frac{2}{7}})$ D. $f(3^{-\frac{2}{7}})>f(2^{-\frac{7}{3}})>f(\log_3\frac{2}{7})$

E. $f(\log_3\frac{2}{7})>f(3^{-\frac{2}{7}})>f(2^{-\frac{7}{3}})$

第二部份 解答題。

1. 10 3 4

   (a) 2 (4)

   (b) (4)

2. $\alpha, \beta \in (0, \frac{\pi}{2})$ $\tan\alpha = \frac{1}{5}$ $\cos\beta = \frac{3\sqrt{13}}{13}$

   (a) $\tan(\alpha+\beta)$ (4)

   (b) $\cos(\alpha+2\beta)$ (4)

3. $\{a_n\}_{n\geq 1}$ $a_1 = 3$ a_1 a_2 a_5

   (a) $\{a_n\}_{n\geq 1}$ (4)

   (b) S_n $\{a_n\}_{n\geq 1}$ $S_n \geq 12 \quad + 36$ (4)

4. $f(\ ) = a - |\ \ - 3| - |\ \ - 7|$

   (a) $a = 8$ $f(\ ) \geq 0$ (4)

   (b) $g(\ ) = \ f(\ )$ $[-1, 1]$ -1 a (4)

5. $\mathcal{C} : \frac{\ ^2}{a^2} - \frac{\ ^2}{b^2} = 1$ $\frac{\sqrt{10}}{2}$ $A(2\sqrt{2}, 3)$ $\ell : \ = \ + m$ $\mathcal{C}$ P Q $OP \perp OQ$ O

   (a) $\mathcal{C}$ (3)

   (b) m (5)

第一部份 選擇題。

1	B
2	B
3	C
4	C
5	D
6	A
7	D
8	A
9	C
10	E
11	B
12	C
13	A
14	B
15	E

第二部份 **解答題。**

1. X

(a) 4 2 $P(X=2)=\frac{{}_3C_2\cdot{}_7C_2}{{}_{10}C_4}=\frac{3}{10}$ 4 3 $P(X=3)=\frac{{}_3C_3\cdot{}_7C_1}{{}_{10}C_4}=\frac{1}{30}$ 2 $P(X\geq 2)=P(X=2)+P(X=3)=\frac{3}{10}+\frac{1}{30}=\frac{1}{3}$

(b) 4 $P(X=0)=\frac{{}_3C_0\cdot{}_7C_4}{{}_{10}C_4}=\frac{1}{6}$ 4 1 $P(X=1)=\frac{{}_3C_1\cdot{}_7C_3}{{}_{10}C_4}=\frac{1}{2}$ $E(X)=0\cdot P(X=0)+1\cdot P(X=1)+2\cdot P(X=2)+3\cdot P(X=3)=1\cdot\frac{1}{2}+2\cdot\frac{3}{10}+3\cdot\frac{1}{30}=\frac{6}{5}$

2. $\beta\in(0,\frac{\pi}{2})$ $\cos\beta=\frac{3\sqrt{13}}{13}$ $\sin\beta=\sqrt{1-\cos^2\beta}=\frac{2\sqrt{13}}{13}$ $\tan\beta=\frac{2}{3}$

(a) $\tan\alpha=\frac{1}{5}$ $\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=1$

(b) $\alpha,\beta\in(0,\frac{\pi}{2})$ (a) $\alpha+\beta=\frac{\pi}{4}$ $\cos(\alpha+2\beta)=\cos[(\alpha+\beta)+\beta]=\cos(\beta+\frac{\pi}{4})=\frac{\sqrt{2}}{2}(\cos\beta-\sin\beta)=\frac{\sqrt{26}}{26}$

3. (a) d $a_1=3$ $a_2=3+d, a_5=3+4d$ a_1 a_2 a_5 $a_2^2=a_1a_5$ $(3+d)^2=3(3+4d)$ $d=$ $d=$ $a_n=3$ $a_n=3+6(\ \ -1)=6\ \ -3$

(b) S_n S_n (

ii.) $-\dfrac{a-10}{4} \geq 1$ $a \leq 6$ $g(\ )$ $= 1$ $2+(a-10) = a-8 =$

-1 $a = 7$

iii.) $-\dfrac{a-10}{4} \leq -1$ $a \geq 14$ $g(\ )$ $= -1$ $2-(a-10) =$

-1 $a = 13$

$a = 10 \pm 2\sqrt{2}$

5. (a) A $\frac{8}{a^2} - \frac{9}{b^2} = 1$ $e = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{10}}{2}$ $a^2 = 2$

$b^2 = 3$ $\frac{x^2}{2} - \frac{y^2}{3} = 1$

(b) l $\mathcal{C}$ $P(\ _1,\ _1)$ $Q(\ _2,\ _2)$ l $\mathcal{C}$

$3\ ^2 - 2(\ + m)^2 - 6 = 0$ $^2 - 4m\ - 2m^2 - 6 = 0$ $_1 + \ _2 = 4m$

$_1\ _2 = -2m^2 - 6$ $OP \perp OQ$ $_1\ _2 + \ _1\ _2 = 0$ P Q l

$(\ _1 + m)(\ _2 + m) + \ _1\ _2 = 0$ $2\ _1\ _2 + m(\ _1 + \ _2) + m^2 = 0$

$2(-2m^2 - 6) + 4m^2 + m^2 = 0$ $m^2 = 12$ $m = \pm 2\sqrt{3}$

1. Let sets $A = \{ \ : \ ^2 - 3 \ - 4 \le 0\}$, $B = \{ \ : 3 \ + a \ge 0\}$ and $A \cap B = \{ \ : 2 \le \ \le 4\}$, then $a = (\quad)$.

A. -12 B. -6 C. -3 D. 6 E. 12

2. Given that $f(\ ) = f(\ + 1) + 1$ for all real numbers and $f(0) = 16$, the value of $f(15)$ is ( ).

A. 0 B. 1 C. 15 D. 16 E. 17

3. Suppose and satisfy $4 \ + 5 \ = \ (\ + 1) - (\ - 1)(\ - 1)$. If the value of is increased by 4, then the value of is ( ).

A. decreased by 8 B. decreased by 4 C. decreased by 2

D. increased by 4 E. increased by 8

4. The coefficient of in the expansion of $(\sqrt{\ } - 2)^5 (2 \ - 1)^4$ is ( ).

A. -182 B. -178 C. 176 D. 178 E. 184

5. $P(2, 3)$ is a fixed point on a - coordinate plane. M is a moving point such that it maintains a fixed distance from point P. If the locus of M passes through the origin, the equation of the locus of M is ( ).

A. $\ ^2 + \ ^2 - 13 = 0$ B. $\ ^2 + \ ^2 + 4 \ - 6 \ = 0$ C. $\ ^2 + \ ^2 + 4 \ + 6 \ = 0$

D. $\ ^2 + \ ^2 - 4 \ - 6 \ = 0$ E. $\ ^2 + \ ^2 - 4 \ - 6 \ + 13 = 0$

6. $\dfrac{3 \log \frac{1}{2} + \log 16}{\log 4 + \log 5 - 1} = (\quad)$.

A. 1 B. -1 C. 2 D. -2 E. 4

7. The sum of the 2nd term and the 5th term of a geometric sequence is 9, while the sum of the 7th term and 10th term of the sequence is 288, then the sequence of the 20th term is ( ).

A. 32768 B. 65536 C. 131072 D. 262144 E. 524288

8. If the mean of the data set $\{ \ , \ - 3, 2 \ + 5, 4 \ - 4, 5 \ + 10\}$ is 6.8, the median of this set is ( ).

A. 4 B. 5 C. 15 D. 0 E. -1

9. $\sqrt{1+\left(\frac{m^4-1}{2m^2}\right)^2} = (\quad)$.

A. $\frac{m^4+2m+1}{2m^2}$

B. $\frac{m^4-1}{2m^2}$

C. $\frac{m^2}{2}+\frac{1}{2m^2}$

D. $\frac{\sqrt{m^2+1}}{2}$

E. None of the above

10. In the acute triangle $\triangle ABC$, $|AB|=8$, $|AC|=7$, $\sin C=\frac{4\sqrt{3}}{7}$. Then $|BC|=(\quad)$.

A. 6 B. 12 C. 2 D. 3 E. 5

11. A parabola cuts the -axis at $(-2,0)$ and $(6,0)$ and the -axis at $(0,4)$. If $(m,\ )$ is a point lying on the parabola, the maximum value of is $(\quad)$.

A. $\frac{8}{3}$ B. $\frac{16}{3}$ C. 4 D. 8 E. 16

12. In the following interval $(\quad)$, the function $f(\ )=5\cos(\ +\frac{\pi}{3})$ increases monotonically.

A. $\left(0,\frac{\pi}{2}\right)$

B. $\left(\frac{\pi}{2},\pi\right)$

C. $\left(\pi,\frac{3\pi}{2}\right)$

D. $\left(\frac{3\pi}{2},2\pi\right)$

E. $\left(\frac{\pi}{3},\frac{5\pi}{6}\right)$

13. If $\theta\in[0,\pi)$ and $1+\sin\theta-2\cos^2\theta=0$, then $\theta=(\quad)$.

A. $\frac{\pi}{6}$ or $\frac{5\pi}{6}$ B. $\frac{\pi}{3}$ C. $\frac{\pi}{6}$ or $\frac{\pi}{3}$ D. $\frac{\pi}{6}$ or $\frac{\pi}{2}$ E. $\frac{\pi}{3}$ or $\frac{\pi}{2}$

14. If $\ ^2-3\ +1=0$, then $\ ^4+\frac{1}{\ ^4}=(\quad)$.

A. 2 B. 47 C. 49 D. 79 E. 81

15. Let $f(\ )$ be an even function defined on $\mathbb{R}$ and decrease in $(-\infty,0)$. Which of the following is true? $(\quad)$.

A. $f(2^{-\frac{7}{3}})>f(3^{-\frac{2}{7}})>f(\log_3\frac{2}{7})$

B. $f(3^{-\frac{2}{7}})>f(\log_3\frac{2}{7})>f(2^{-\frac{7}{3}})$

C. $f(\log_3\frac{2}{7})>f(2^{-\frac{7}{3}})>f(3^{-\frac{2}{7}})$

D. $f(3^{-\frac{2}{7}})>f(2^{-\frac{7}{3}})>f(\log_3\frac{2}{7})$

E. $f(\log_3\frac{2}{7})>f(3^{-\frac{2}{7}})>f(2^{-\frac{7}{3}})$

1. There are 3 defective products among the 10 products. Now 4 products are randomly selected.

   (a) Find the probability that at least 2 defective products are selected. (4 marks)

   (b) Find the expected number of defective products selected. (4 marks)

2. Given that $\alpha, \beta \in (0, \frac{\pi}{2})$, $\tan \alpha = \frac{1}{5}$, $\cos \beta = \frac{3\sqrt{13}}{13}$.

   (a) Find the value of $\tan(\alpha + \beta)$. (4 marks)

   (b) Find the value of $\cos(\alpha + 2\beta)$. (4 marks)

3. In the arithmetic series $\{a_n\}_{n \geq 1}$, $a_1 = 3$. a_1, a_2 and a_5 form a geometic series.

   (a) Find the general term for $\{a_n\}_{n \geq 1}$. (4 marks)

   (b) Let S_n be the th partial sum of $\{a_n\}_{n \geq 1}$. Is there a positive integer such that $S_n \geq 12\ \ + 36$? If yes, find the smallest value of such . If no, give your reason. (4 marks)

4. Suppose the function $f(\ \ ) = a - |\ \ - 3| - |\ \ - 7|$.

   (a) If $a = 8$, solve the inequality $f(\ \ ) \geq 0$. (4 marks)

   (b) If the function $g(\ \ ) = \ \ f(\ \ )$ has its minimum value -1 in the closed interval $[-1, 1]$, find the value of a. (4 marks)

5. Suppose that the eccentricity of the hyperbolic $\mathcal{C} : \frac{\ ^2}{a^2} - \frac{\ ^2}{b^2} = 1$ is $\frac{\sqrt{10}}{2}$. The point $A(2\sqrt{2}, 3)$ lies on the hyperbolic $\mathcal{C}$. The straight line $\ell : \ \ = \ \ + m$ intersects $\mathcal{C}$ at points P and Q and $OP \perp OQ$, where Q is the origin of the coordinate system.

   (a) Find the equation for $\mathcal{C}$. (3 marks)

   (b) Find the value of m. (5 marks)

Suggested Answer

Question Number	Best Answer
1	B
2	B
3	C
4	C
5	D
6	A
7	D
8	A
9	C
10	E
11	B
12	C
13	A
14	B
15	E

1. Let the number of defective items selected be X.

   (a) The probability of selecting exactly 2 defective items out of 4 is $P(X = 2) = \frac{{}_3C_2 \cdot {}_7C_2}{{}_{10}C_4} = \frac{3}{10}$, and the probability of selecting exactly 3 defective items out of 4 is $P(X = 3) = \frac{{}_3C_3 \cdot {}_7C_1}{{}_{10}C_4} = \frac{1}{30}$. Therefore, the probability of selecting at least 2 defective items is $P(X \geq 2) = P(X$0$= 9 \quad 8$

(b) $f(\ )$ is defined on the closed interval $[-1, 1]$. Thus $f(\ ) = a - (3 - \ ) - (7 - \ ) = 2\ \ + (a - 10)$ and $g(\ ) = \ \ f(\ ) = 2\ ^2 + (a - 10)\ $. The axis of symmetry for $g(\ )$ is $\ _0 = -\dfrac{a - 10}{4}$.

i.) If $-\dfrac{a-10}{4} \in [-1, 1]$, i.e., $a \in [6, 14]$, then the minimum value of $g(\ )$ is $-\dfrac{(a-10)^2}{8} = -1$, giving $a = 10 \pm 2\sqrt{2}$.

ii.) If $-\dfrac{a-10}{4} \geq 1$, i.e., $a \leq 6$, then $g(\ )$ achieves its minimum value at $\ = 1$. That is, $2 + (a - 10) = a - 8 = -1$, and $a = 7$. This does not satisfy the condition.

iii.) If $-\dfrac{a-10}{4} \leq -1$, i.e., $a \geq 14$, then $g(\ )$ achieves its minimum value at $\ = -1$. That is $2 - (a - 10) = -1$, and $a = 13$. This does not satisfy the condition.

In conclusion, $a = 10 \pm 2\sqrt{2}$.

5. (a) Point A lies on the hyperbola, so $\frac{8}{a^2} - \frac{9}{b^2} = 1$. Also, since the eccentricity $e = \frac{\sqrt{a^2+b^2}}{a} = \frac{\sqrt{10}}{2}$, it is easy to find $a^2 = 2$ and $b^2 = 3$. Therefore, the equation of the hyperbola is $\frac{x^2}{2} - \frac{y^2}{3} = 1$.

(b) Suppose the intersection points of the line l and the hyperbola $\mathcal{C}$ are $P(\ _1, \ _1)$ and $Q(\ _2, \ _2)$. By solving the equations of line l and hyperbola $\mathcal{C}$ together, we have $3\ ^2 - 2(\ \ + m)^2 - 6 = 0$, i.e., $\ ^2 - 4m\ \ - 2m^2 - 6 = 0$. According to Vieta's formulas, $\ _1 + \ _2 = 4m$ and $\ _1\ _2 = -2m^2 - 6$. Since $OP \perp OQ$, we have $\ _1\ _2 + \ _1\ _2 = 0$. Noting that points P and Q lie on the line l, we get $(\ _1 + m)(\ _2 + m) + \ _1\ _2 = 0$. Simplifying, we obtain $2\ _1\ _2 + m(\ _1 + \ _2) + m^2 = 0$. From Vieta's formulas, we have $2(-2m^2 - 6) + 4m^2 + m^2 = 0$, which gives $m^2 = 12$, so $m = \pm 2\sqrt{3}$.