澳門四高校聯合入學考試（語言科及數學科）

Joint Admission Examination for Macao Four Higher Education Institutions

(Languages and Mathematics)

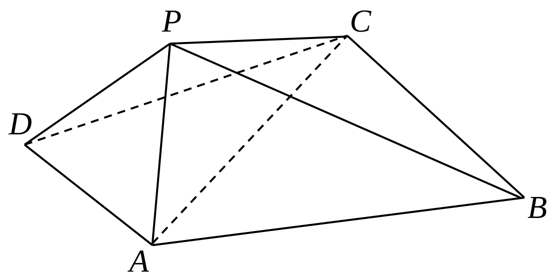
2024 年試題及參考答案

2024 Examination Paper and Suggested Answer

數學附加卷 Mathematics Supplementary Paper

Answer any 3 questions, each carries 20 marks. Write down the answers on the 3 lined pages following each question.

1.



AB A B B

A $\frac{}{0}$ A $\frac{}{4}$ 9 AB 0

(a) B A (6)

(b) AB (6)

(c) B A B

[: B] (8)

In the above figure, AB is a quadrilateral, A is an equilateral triangle, B is a right-angled isosceles triangle with hypotenuse B. A $\frac{}{0}$, A $\frac{}{4}$, 9 and AB 0.

(a) Show that B and A are perpendicular. (6 marks)

(b) Find the volume of the triangular pyramid AB. (6 marks)

(c) Find the cosine of the dihedral angle between plane B and plane A B.

[Hint. Let be the mid-point of B .] (8 marks)

2. (a) 1 ◦

(i) 0 — 7 2 4 , 9 ◦ (2)

(ii) 9 0 ◦ (4)

(iii) 0 ◦ (3)

(iv) (ii) (iii) 0 (2)

(v) (1)

(b) *k* — *i* — 0*i*

1 *k* (8)

(a) The slant height of a right circular cone is 1 m. Suppose its base radius is m and its volume is 1.

(i) Show that 0 — 7 2 4 , 9. (2 marks)

(ii) Find the local maximum and local minimum values of 0 when 9. (4 marks)

(iii) Find the inflection point(s) of the curve 0 . (3 marks)

(iv) Using the results in (ii) (iii), sketch the curve 0 (2 marks)

(v) What is the maximum possible volume of the cone? (1 mark)

(b) Let *k* be a positive constant. Suppose, in the first quadrant, the area of the region bounded by the line and the two curves — *i* and — 0*i* is 1. Find the value of *k*. (8 marks)

3. H: 0 — 2 9 $\overline{3}$

0 0 k

(a) 0 k 0 2 0 0 $\overline{3}k$ 0 $3k$ 0 2 (2)

(b) k (4)

(c) M k M M (6)

(d) k $\overline{3}$ M

[: M M] (8)

Given a hyperbola H: 0 — 2 9. A non-vertical line L passing through the point $\overline{3}$ interests with H at two distinct points and 0 0. Let k be the slope of .

(a) Show that and 0 satisfy the equation

k 0 2 0 0 $\overline{3}k$ 0 $3k$ 0 2 . (2 marks)

(b) Find the range of k (4 marks)

(c) Let M be the origin. Find the value(s) of k such that M M . (6 marks)

(d) Suppose k $\overline{3}$. Find the area of the triangle M .

[Hint. The segment M divides the triangle M into two triangles.] (8 marks)

4. $\overline{\quad}$ 9

(a) (i) *am* $\frac{\bar{1}}{\quad}$ (4)

(ii) $\frac{\bar{1}}{\quad}$ ^{04} (4)

(b) am g

am $\frac{}{0}$ ^{9} g $\frac{}{0}$ ^{9}

g ^{0} am ^{1} $\frac{}{4}$ 0 am am 1 am 3 (8)

(c) 0 am am 1 am 3 (4)

Let $\overline{\quad}$ 9.

(a) (i) Express $\frac{\bar{1}}{\quad}$ in polar form *am* , where and . (4 marks)

(ii) Find $\frac{\bar{1}}{\quad}$ ^{04}. Express your answer in the form , where and are real numbers. (4 marks)

(b) Let am g . Using theorem, show that for any positive integer ,

am $\frac{}{0}$ ^{9} and g $\frac{}{0}$ ^{9} .

Deduce that g ^{0} am ^{1} $\frac{}{4}$ 0 am am 1 am 3 . (8 marks)

(c) Find the general solution of the equation 0 am am 1 am 3 . (4 marks)

5. (a) a a a 9 9 a 9 (7)

(b) k x y z :

$$x \quad 9$$
$$C \;, \; i \quad 3 \quad x \quad 1$$
$$i \quad x \quad 9$$

(i) k (E) (5)

(ii) i 1 (E) (4)

(c) a

$$x \quad 9$$
$$1 \quad 3 \quad x \quad 1$$
$$1 \quad x \quad 9$$
$$\lceil 0 \quad 0 \; - \quad 1x$$

a (4)

(a) Factorize the determinant a a a 9 9 a 9 . (7 marks)

(b) Let *k* be a constant. Given the system of equations with unknowns *x*, *y* and *z*:

$$x \quad 9$$
$$C \;, \; i \quad 3 \quad x \quad 1 \, .$$
$$i \quad x \quad 9$$

(i) Find the range of *k* such that (*E*) has a unique solution. (5 marks)

(ii) Suppose i 1. Find the general solution of (*E*). (4 marks)

(c) Find the maximum value of *a* such that the system of equations

$$x \quad 9$$
$$1 \quad 3 \quad x \quad 1$$
$$1 \quad x \quad 9$$
$$\lceil 0 \quad 0 \; - \quad 1x$$

has a solution. For this value of *a*, solve the system of equations. (4 marks)

1. (a)

$A \quad \frac{\leq}{\text{r l} \quad A\leq} \quad \frac{}{\text{r l} -} \quad \bar{1}$ (1)

$B^{0} \quad A^{0} \quad 2 \quad AB^{0}$

$B \quad A \quad \frac{-}{0}$ (2)

$B \quad A \quad B \quad B \quad A \quad B \quad A$

(b) (2) g $B\,A \quad \frac{\leq B}{B\,A} \quad \frac{-}{0} \quad B\,A \quad \frac{-}{4}$

$A \quad A \quad \frac{-}{1} \quad B\,A \quad B\,A \quad A \quad \frac{-}{0}$

(1) $A \quad A \quad \bar{1} \quad DCB \quad \frac{-}{0} \; B\,A \quad A \quad \bar{1}$

$AB \quad \frac{-}{1} \quad B\,A \quad \frac{\bar{1}}{1}$

(c) $M \quad DP \quad B \quad B$

(1)

$A \quad \overline{ 0 A^{0}} \quad 0 \quad AB$ 1

$M \quad DP \quad A \quad B$

A

$M \quad DP \quad \frac{-}{0} B \quad \frac{-}{0} \quad \overline{B^{0} 0} \quad \frac{\bar{0}}{0}$

$\overline{ 0 0} \quad \frac{\bar{0}}{0}$

$A \quad (3) \quad A \quad \overline{A^{0} 0} \quad \frac{\bar{2}}{0}$

am $A \quad \frac{\leq \quad A \;\; 9 \leq A}{0 \;\; \leq \;\; A} \quad \frac{\bar{5}}{5}$

[: $A \quad \frac{-}{0}$ am $A \quad \frac{\leq}{A}$]

2. (a)(i) $0 \;\; \bar{1} \;\; 0 \;\; 0 \;\; 9 \quad 0 \quad \frac{-}{7}^{2} \; 9 \quad 0 \quad \frac{-}{7}^{2} \quad 4$

(ii) $\frac{b}{b} \quad \frac{0}{7} \quad 0^{1} \quad 1^{3} \quad 9 \;\; \frac{b}{b} \quad \Rightarrow 0^{1} \quad 1^{3} \quad \Rightarrow \quad \frac{\bar{4}}{1}$

$\frac{\bar{4}}{1} \quad \frac{b}{b} \quad 0$

$\frac{\bar{4}}{1} \quad 9 \;\; \frac{b}{b} \quad 0$

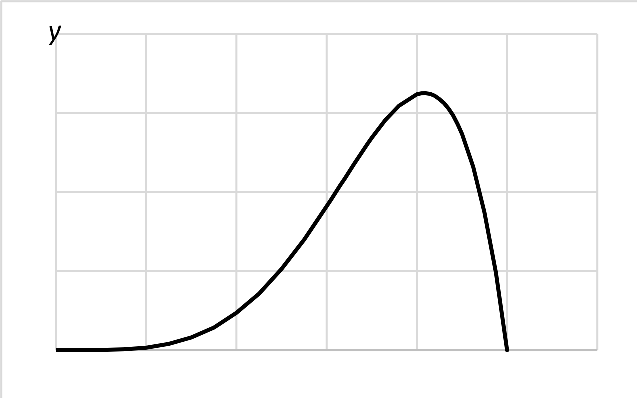
$0 \quad \frac{\bar{4}}{1} \quad \frac{2}{021}$

(iii) $\frac{b}{b} \quad \frac{0}{1} \quad 0^{0} \quad 3^{2} \quad 9 \;\; \frac{b}{b} \quad \Rightarrow 0^{0} \quad 3^{2} \quad \Rightarrow \quad \frac{-}{3}$

$\frac{-}{3} \quad \frac{b}{b} \quad \frac{-}{3} \quad 9 \;\; \frac{b}{b}$

$\frac{-}{3} \quad \frac{2}{153} \quad 0$

(iv)



(v) $\frac{0\ \overline{1}}{05}$

(b) $\overline{i}$ i $\overline{0i}$ $0i$

i 0 0 $0i$ 0 1 i 0 1 $0i$

$\frac{}{i}$ $\frac{}{0i}b$ i $\frac{}{0i}b$ 9 $\frac{}{4i}$ $\frac{}{0}$ $\frac{}{4i}$ i 9

$\frac{i\ 0}{4}$ $0i\ 0$ $\frac{2i\ 0}{1}$ $\frac{i\ 0}{0}$ $\frac{i\ 0}{4}$ 9

i $\overline{0}$ i

3. (a) 0 $\overline{2}$ 9 k $\overline{3}$ 2 0 k 0 $\overline{3}$ 0 2

k 0 2 0 0 $\overline{3}k$ 0 3k 0 2 (1)

(b) (1) k 0 2 (1)

0 k 0

0 $\overline{3}k$ 0 0 2 k 0 2 3k 0 2 (2)

(2) 42k 0 42 k k 0

(c) (1)

0 $\frac{0\ \overline{3}k\ 0}{k\ 0\ \ 2}$ 0 $\frac{3k\ 0\ \ 2}{k\ 0\ \ 2}$ 1

M M —

(c)

0 am am 1 am 3 94 g 0 am 1

g am

i 0 i i

0

5. (a)

a a

a a a a a

9 9 a 9 9 9 a 9

a

a 9 9 9

9 9 a 9

a

a 9

9 a

a 9 a a

a 0 0 a^0 a a

9 9 9

(b)(i) (E) i 3 9 i 1

9 i 9

x 9

(ii) i 1 1 3 x 1

1 x 9

x 9

1 3 x 1 9 0r r x r r

(c) r 0 9 0r 0 $\bar{r}$ 1r 1 9 $\bar{r}$ 0

3 1 r 9 9 9 x 9

Suggested Answers:

1. (a) It follows from

$$A \quad \frac{\leq}{\text{r l} \quad A\leq} \quad \frac{}{\text{r l} -} \quad \overline{1} \qquad (1)$$

that B^0 A^0 2 AB^0. Hence,

$$B \quad A \quad \frac{-}{0}. \qquad (2)$$

From B A and B , we get B n l c A. Hence B A.

(b) From (2), we have g BA $\frac{\leq B}{BA}$ $\frac{-}{0}$. Hence, BA $\frac{-}{4}$.

Since A is equilateral, A $\frac{-}{1}$. Hence, BA BA A $\frac{-}{0}$.

Using (1), as A A $\overline{1}$, the area of DCB is $\frac{-}{0}$ BA A $\overline{1}$.

The volume of AB is $\frac{-}{1}$ pc md BA $\frac{\overline{1}}{1}$.

(c) As M is the mid-point of DP and B , we get B .

Using (1), we get

$$A \quad \overline{\quad 0 \quad A^0} \quad 0 \quad AB \qquad 1$$

With M is the mid-point of DP, we get A B . Hence, the required dihedral angle is A .

As M is the mid-point of DP, we get $\frac{-}{0}$ B $\frac{-}{0}$ $\overline{B^0 \quad 0}$ $\frac{\overline{0}}{0}$.

As is a right angle, we get $\overline{0 \quad 0}$ $\frac{\overline{0}}{0}$.

As A is a right angle, using (3), we get A $\overline{A^0 \quad 0}$ $\frac{\overline{2}}{0}$.

Hence, am A $\frac{\leq \quad A \;\; 9 \leq A}{0 \quad \leq \quad A}$ $\frac{\overline{5}}{5}$.

[Remark: One may prove A $\frac{-}{0}$. Then am A $\frac{\leq}{A}$.]

2. (a)(i) Suppose the height of the right circular cone is m.

pm $_0$ $\overline{1}$ $_0$ ^{0} 9 , we get ^{0} $\frac{-}{7}$ ^{2} 9 ^{0} $\frac{-}{7}$ ^{2} ^{4} .

(ii) $\frac{b}{b}$ $\frac{0}{7}$ 0^1 1^3 f cl 9 $\frac{b}{b}$ $\Rightarrow 0^1$ 1^3 $\Rightarrow$ $\frac{\overline{4}}{1}$

When $\frac{\overline{4}}{1}$, $\frac{b}{b}$. So, ^{0} is increasing.

When $\frac{\overline{4}}{1}$ 9 $\frac{b}{b}$. So, ^{0} is decreasing.

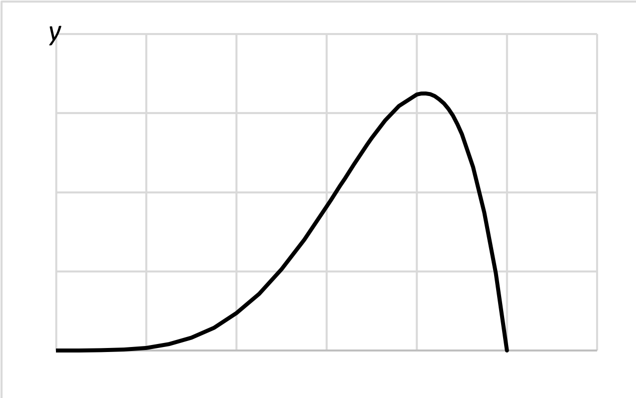
Hence, ^{0} $\frac{\overline{4}}{1}$ $\frac{2}{021}$ is a local maximum value.

(iii) $\frac{b}{b}$ $\frac{0}{1}$ 0^0 3^2 f cl 9 $\frac{b}{b}$ $\Rightarrow 0^0$ 3^2 $\Rightarrow$ $\frac{\overline{\quad}}{3}$

When $\frac{\overline{\quad}}{3}$, $\frac{b}{b}$ f cl $\frac{\overline{\quad}}{3}$ 9, $\frac{b}{b}$.

Hence, $\frac{\overline{\quad}}{3}$ $\frac{2}{153}$ is an inflection point of the curve ^{0} .

(iv)



(v) $\frac{0\ \overline{1}}{05}$ 1

(b) Solving $\frac{}{i}$, we get or i. Solving $\frac{}{0i}$, we get or $0i$.

As the area of the bounded region is 1, we get

$$\frac{i}{i}\ \frac{0}{0i}b \quad \frac{0}{0i}b \quad 9 \quad \frac{1}{4i} \quad \frac{0}{0} \quad \frac{1}{4i} \quad 9$$

$$\frac{i^0}{4} \quad 0i^0 \quad \frac{2i^0}{1} \quad \frac{i^0}{0} \quad \frac{i^0}{4} \quad 9$$

$$i \quad \overline{0} \quad i$$

3. (a) pm $\frac{}{k}$ 9 $\frac{}{\overline{3}}$, we get 2 ^{0} k^0 $\overline{3}^0$ 2. That is,

$$k^0 \quad 2 \quad ^0 \quad 0\ \overline{3}k^0 \quad 3k^0 \quad 2 \quad . \qquad (1)$$

(b) As the non-vertical line and have two distinct intersection points, (1) has two distinct real roots. So, k^0 2 l b rf c bg apg g l r md 9 g gecprf l 0. That is, k 0 and

$$0\ \overline{3}k^0\ ^0 \quad 2\ k^0 \quad 2 \quad 3k^0 \quad 2 \qquad (2)$$

From (2), we get $42k^0$ 42 which is true for all *m*. Thus, the range of k is 0 .

(c) From (1), we get

$$_0 \quad \frac{0\ \overline{3}k^0}{k^0 \quad 2} \quad \text{l b} \quad _0 \quad \frac{3k^0 \quad 2}{k^0 \quad 2} \qquad 1$$

Hence,

$$M \quad M \quad — \frac{0}{0} \quad 9$$

$$k \quad \overline{3} \quad k \quad {}_0 \quad \overline{3} \quad {}_0$$

$$k^{0} \quad 9 \quad {}_0 \quad \overline{3}k^{0} \quad {}_0 \quad 3k^{0}$$

$$k^{0} \quad 9 \quad \frac{3k^{0} \quad 2}{k^{0} \quad 2} \quad \overline{3}k^{0} \quad \frac{0 \quad \overline{3}k^{0}}{k^{0} \quad 2} \quad 3k^{0}$$

$$k^{0} \quad 9 \quad 3k^{0} \quad 2 \quad 9 \quad k^{2} \quad 3k^{0} \quad k^{0} \quad 2$$

$$99k^{0} \quad 2$$

$$k \quad \frac{0 \quad \overline{99}}{99}$$

(d) Substituting k $\overline{3}$ into (3), we get ${}_0$ 9 $\overline{3}$ and ${}_0$ 0 . So, and ${}_0$ are positive numbers. Hence, we know that points A and B are on the same branch. We may assume and ${}_0$. Then,

$$\text{pc} \quad \text{md} \quad M \quad \text{pc} \quad \text{md} \quad M \quad \text{pc} \quad \text{md} \quad M$$

$$\frac{9}{0} \, M \quad \frac{9}{0} \, M \quad {}_0 \quad \frac{9}{0} \, \overline{3} \quad {}_0$$

By direct calculation,

$${}_0{}^{0} \quad {}_0{}^{0} \quad 2 \quad {}^{0}$$

$$\overline{3} \quad \overline{3} \quad \overline{3} \quad {}_0 \quad \overline{3} \quad {}^{0} \quad 2 \quad \overline{3} \quad \overline{3} \quad \overline{3} \quad {}_0 \quad \overline{3}$$

$$\overline{3} \quad {}_0 \quad 9 \quad {}^{0} \quad 0 \quad {}_0 \quad \overline{3} \quad {}_0 \quad 3$$

$$9 \quad 0$$

The pc md M is 0 $\overline{4}$

4. (a) (i) $\frac{\overline{1}}{} \quad \frac{0 \text{ am} - \quad \text{g} -}{\overline{0} \text{ am} - \quad \text{g} -} \quad \overline{0} \text{ am} \frac{}{4} \frac{}{2} \quad \text{g} \frac{}{4} \frac{}{2} \quad \overline{0} \text{ am} \frac{}{0} \quad \text{g} \frac{}{0}$

(ii)

$$\frac{\overline{1}}{}^{04} \quad \overline{0}^{04} \text{ am} \frac{}{0} \quad \text{g} \frac{}{0}^{04}$$

$$0^{1} \text{ am} \frac{04}{0} \quad \text{g} \frac{04}{0}$$

$$0^{1} \text{ am} \frac{}{4} \quad \text{g} \frac{}{4}$$

$$0^{0} \quad \overline{1} \quad 0^{0}$$

(b) The results follow from the sum and difference of

$$\text{am} \quad \text{g} \quad \text{and} \quad {}^{9} \quad \text{am} \quad \text{g} \quad \text{am} \quad \text{g} \quad .$$

$$\text{g}^{0} \quad \text{am}^{1} \quad \frac{}{0} \quad {}^{9} \quad {}^{0}\frac{}{0} \quad {}^{9} \quad {}^{1}$$

$$\frac{}{10} \quad {}^{0} \quad 0 \quad {}^{90} \quad {}^{1} \quad 1 \quad 1 \quad {}^{9} \quad {}^{91}$$

$$\frac{}{10} \quad {}^{3} \quad {}^{93} \quad {}^{1} \quad {}^{91} \quad 0 \quad {}^{9}$$

$$\frac{}{4} \, 0 \text{ am} \quad \text{am } 1 \quad \text{am } 3$$

(c)

0 am am 1 am 3 94 g 0 am 1

g mp am

i mp $\frac{}{0}$ i i g l g rcecp

$\frac{}{0}$ g l g rcecp

5. (a)

a a

a a a a a

9 9 a 9 9 9 a 9

a

a 9 9 9

9 9 a 9

a

a 9

9 a

a 9 a a

a 0 0 a^0 a a

(b)(i) (E) has a unique solution if and only if 9 9 9 / i 3 9 / 9 i 9, that is, i 1.

(ii) When i 1, the system of equations becomes x 9 / 1 3 x 1 / 1 x 9.

Solving x 9 / 1 3 x 1, we get 9 0r, r x r r .

This solution also satisfies the third equation and hence is the general solution of the system of equations.

(c) Suppose the system of equations has a solution. Then there exists r such that

0 9 0r 0 $\bar{r}$ 1r , that is, 1 9 $\bar{r}$ 0 Hence, we know that the maximum value of is 3. When 1, we have r 9 and the solution of the system of equations is 9 9 x 9