澳門四高校聯合入學考試（語言科及數學科）

Joint Admission Examination for Macao Four Higher Education Institutions (Languages and Mathematics)

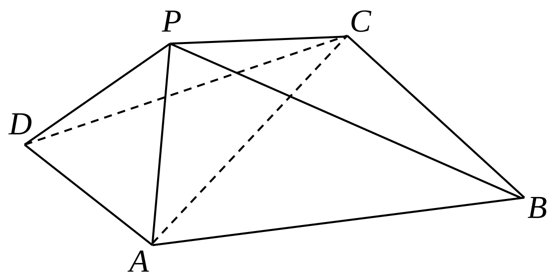
2024 年試題及參考答案
2024 Examination Paper and Suggested Answer

數學附加卷 Mathematics Supplementary Paper

3

Answer any 3 questions, each carries 20 marks. Write down the answers on the 3 lined pages following each question.

1.



A M A MA

M $-$ M $\overline{}$ 3 M A 9

(a) A M (6)

(b) M A (6)

(c) MA MA

[: AM] (8)

In the above figure, A is a quadrilateral, is an equilateral triangle, M A is a right-angled isosceles triangle with hypotenuse MA. M $-$, M $\overline{}$ 3,

M and A 9.

(a) Show that A and M are perpendicular. (6 marks)

(b) Find the volume of the triangular pyramid M A. (6 marks)

(c) Find the cosine of the dihedral angle between plane MA and plane MA.

[Hint. Let be the mid-point of AM.] (8 marks)

2. (a) 1 ◦

(i) $\frac{}{6}$ ^{1} ^{3} , ◦ (2)

(ii) □ ◦ (4)

(iii) ◦ (3)

(iv) (ii) (iii) (2)

(v) (1)

(b) k $\frac{}{h}$ $\frac{}{h}$

1 k (8)

(a) The slant height of a right circular cone is 1 m. Suppose its base radius is m and its volume is □ ^{0}.

(i) Show that $\frac{}{6}$ ^{1} ^{3} , . (2 marks)

(ii) Find the local maximum and local minimum values of □when □ □□□□□□□□□ □ . (4 marks)

(iii) Find the inflection point(s) of the curve . (3 marks)

(iv) Using the results in (ii) (iii), sketch the curve□ (2 marks)

(v) What is the maximum possible volume of the cone? (1 mark)

(b) Let k be a positive constant. Suppose, in the first quadrant, the area of the region bounded by the line and the two curves $\frac{}{h}$ and $\frac{}{h}$ is 1. Find the value of k. (8 marks)

3. H: $\frac{}{1}$ M $\overline{2}$? E

(a) ? 1 9 $\overline{2}$ 2 1 ? (2)

(b) (4)

(c) L L L (6)

(d) $\overline{2}$ L

[: LM L] (8)

Given a hyperbola H: $\frac{}{1}$. A non-vertical line L passing through the point M $\overline{2}$ interests with H at two distinct points and . Let be the slope of .

(a) Show that and satisfy the equation

1 9 $\overline{2}$ 2 1 . (2 marks)

(b) Find the range of , (4 marks)

(c) Let L be the origin. Find the value(s) of such that L L . (6 marks)

(d) Suppose $\overline{2}$. Find the area of the triangle L .

[Hint. The segment LM divides the triangle L into two triangles.] (8 marks)

4. —

(a) (i) o k $\frac{\bar{0}7}{7}$ o (4)

(ii) $\left(\frac{\bar{0}7}{7}\right)^3$ ⍰ (4)

(b) l p pf k

l pk – ^{k} ^{k} pf k — ^{k} ^{k}

pf l p^0 $\frac{}{3}$ 9 l p l p0 l p2 (8)

(c) 9 l p l p0 l p2 (4)

Let —.

(a) (i) Express $\frac{\bar{0}7}{7}$ in polar form o k , where o and . (4 marks)

(ii) Find $\left(\frac{\bar{0}7}{7}\right)^3$. Express your answer in the form , where and are real numbers. (4 marks)

(b) Let l p pf . Using theorem, show that for any positive integer k,

l pk – ^{k} ^{k} and pf k — ^{k} ^{k} .

Deduce that pf l p^0 $\frac{}{3}$ 9 l p l p0 l p2 . (8 marks)

(c) Find the general solution of the equation 9 l p l p0 l p2 . (4 marks)

5. (a) (7)

(b) k x y z :

B h 2 0
h

(i) k (E) (5)

(ii) h 0 (E) (4)

(c) a

0 2 0
0
9 9 0

a (4)

(a) Factorize the determinant . (7 marks)

(b) Let k be a constant. Given the system of equations with unknowns x, y and z:

B h 2 0
h

(i) Find the range of k such that (E) has a unique solution. (5 marks)

(ii) Suppose h 0. Find the general solution of (E). (4 marks)

(c) Find the maximum value of a such that the system of equations

0 2 0
0
9 9 0

has a solution. For this value of a, solve the system of equations. (4 marks)

1. (a)

$\frac{\neq M}{M \geq \neq}$ — $\overline{0}$ (1)

A 1 A

A – (2)

A A M A M A M

(b) (2) pf A $\frac{\neq A}{A \geq}$ – A $\frac{-}{3}$

$\frac{-}{0}$ A A –

(1) $\overline{0}$ DCB – A $\overline{0}$

M A $\frac{-}{0}$ M A $\frac{\overline{0}}{0}$

(c) M DP A M AM

(1)

M M 9 A 0

M DP AM

M DP M – AM – A M

M M M

M (3) M M $\frac{\overline{1}}{}$

l p $\frac{\neq 7 \geq \neq\geq}{\neq \geq}$ $\frac{\overline{4}}{4}$

[: – l p $\frac{\neq}{\geq}$]

2. (a)(i) $\frac{-}{0}$ $\frac{}{6}$ 1 $\frac{}{6}$ 1 3

(ii) $\frac{a}{a}$ $\frac{}{6}$ 9 0 0 2 $\frac{a}{a}$ 9 0 0 2 $\frac{\overline{3}}{0}$

$\frac{\overline{3}}{0}$ $\frac{a}{a}$

$\frac{\overline{3}}{0}$ $\frac{a}{a}$

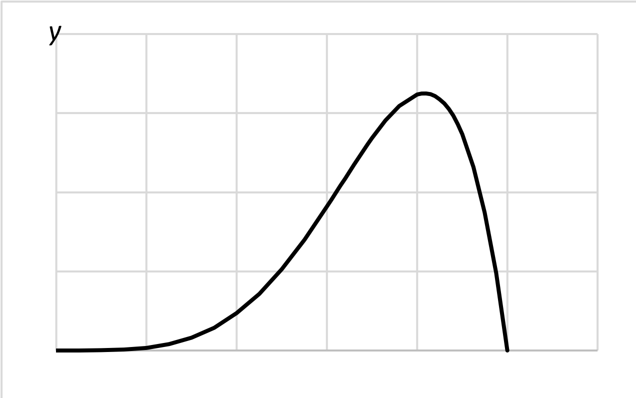
$\frac{\overline{3}}{0}$ $\frac{1}{10}$

(iii) $\frac{a}{a}$ $\frac{}{0}$ 9 2 1 $\frac{a}{a}$ 9 2 1 $\frac{}{2}$

$\frac{}{2}$ $\frac{a}{a}$ $\frac{}{2}$ $\frac{a}{a}$

$\frac{}{2}$ $\frac{1}{042}$

(iv)



(v) $\frac{\overline{0}}{4}$

(b) $\frac{}{h}$ h $\frac{}{h}$ $9h$

$\frac{h}{h}$ $\frac{}{9h}a$ $\frac{h}{h}$ $\frac{}{9h}a$ $\frac{0}{3h}^{h}$ $\frac{}{9}$ $\frac{0}{3h}^{h}_{h}$

$\frac{h}{3}$ $9h$ $\frac{1h}{0}$ $\frac{h}{9}$ $\frac{h}{3}$

h $\overline{9}$ h

3. (a) $\frac{}{1}$ $\frac{}{\overline{2}}$ 1 $\overline{2}$ 1

1 9 $\overline{2}$ 2 1 (1)

(b) E (1) 1 (1)
0 9

9 $\overline{2}$ 1 1 2 1 (2)
(2) 31 31 9

(c) (1)

$\frac{9\ \overline{2}}{1}$ $\frac{2\quad 1}{1}$ 0

L L —

(c)

9 l p l p0 l p2 3 pf $l\,p^0$

pf l p

h $\overline{9}$ h h

$\dfrac{k}{9}$ k

5. (a)

(b)(i) (E) h 2 h 0
h

(ii) h 0 0 2 0
0
0 2 0 9

(c) 9 9 9 $^{-}$ 0 0 $^{-}$
3 0

Suggested Answers:

1. (a) It follows from

$$\frac{\neq M}{M\geq\neq} \quad \overline{} \quad \overline{0} \qquad (1)$$

that A 1 A . Hence,

$$A \quad -. \qquad (2)$$

From A and A M, we get A mi b M . Hence A M .

(b) From (2), we have pf A $\frac{\neq A}{A\geq}$ –. Hence, A $\frac{}{3}$.

Since is equilateral, $\frac{}{0}$. Hence, A A –.

Using (1), as $\overline{0}$, the area of DCB is – A $\overline{0}$.

The volume of M A is $\frac{}{0}$ M ob 1 d A $\frac{\overline{0}}{0}$.

(c) As M is the mid-point of DP and A M, we get AM.
Using (1), we get

M M 9 A , 0

With M is the mid-point of DP, we get AM. Hence, the required dihedral angle is .

As M is the mid-point of DP, we get M – AM – A M —.

As M is a right angle, we get M M —.

As M is a right angle, using (3), we get M M $\frac{\overline{1}}{}$.

Hence, l p $\frac{\neq \; 7 \; \geq \; \neq\geq}{\neq \; \geq}$ $\frac{\overline{4}}{4}$.

[Remark: One may prove –. Then l p $\frac{\neq}{\geq}$.]

2. (a)(i) Suppose the height of the right circular cone is m.

ol $\overline{0}$, we get $\frac{}{6}$ 1 $\frac{}{6}$ 1 3 .

(ii) $\frac{a}{a}$ $\frac{}{6}$ 9 0 0 2 , eb $\frac{a}{a}$ 9 0 0 2 $\frac{\overline{3}}{0}$

When $\frac{\overline{3}}{0}$, $\frac{a}{a}$. So, is increasing.

When $\frac{\overline{3}}{0}$ $\frac{a}{a}$. So, is decreasing.

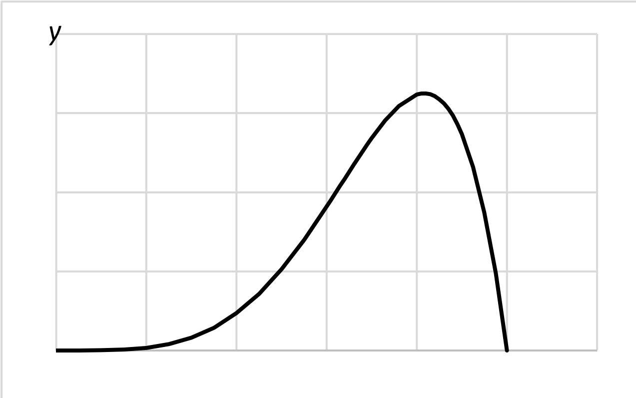
Hence, $\frac{\overline{3}}{0}$ $\frac{1}{10}$ is a local maximum value.

(iii) $\frac{a}{a}$ $\frac{}{0}$ 9 2 1 , eb $\frac{a}{a}$ 9 2 1 $\frac{\overline{}}{2}$

When $\frac{\overline{}}{2}$, $\frac{a}{a}$, eb $\frac{\overline{}}{2}$, $\frac{a}{a}$.

Hence, $\frac{\overline{}}{2}$ $\frac{1}{042}$ is an inflection point of the curve .

(iv)



(v) $\frac{\bar{0}}{4}$ 0

(b) Solving $\frac{}{h}$, we get or h. Solving $\frac{}{h}$, we get or $9h$.

As the area of the bounded region is 1, we get

$$\frac{h}{h} \quad \frac{}{9h} a \quad \frac{h}{} \quad \frac{}{9h} a \quad \frac{0}{3h}^{h} \quad \frac{}{9} \quad \frac{0}{3h}^{h}_{h}$$

$$\frac{h}{3} \quad 9h \quad \frac{1h}{0} \quad \frac{h}{9} \quad \frac{h}{3}$$

$$h \quad \bar{9}, \quad h$$

3. (a) ol $\frac{}{1}$ $\frac{}{\bar{2}}$, we get 1 $\bar{2}$ 1. That is,

$$1 \quad 9 \ \bar{2} \quad 2 \quad 1 \quad . \qquad (1)$$

(b) As the non-vertical line and E have two distinct intersection points, (1) has two distinct real roots. So, 1 ae bafp of f 1 c fp fddbde 0. That is, 9 and

$$9 \ \bar{2} \quad 1 \quad 1 \quad 2 \quad 1 \quad , \qquad (2)$$

From (2), we get 31 31 which is true for all m. Thus, the range of is 9 .

(c) From (1), we get

$$\frac{9 \ \bar{2}}{1} \quad a \quad \frac{2 \quad 1}{1}, \quad 0$$

Hence,

$L \quad L \quad — \quad —$

$\bar{2} \quad \bar{2}$

$\bar{2} \quad 2$

$\frac{2 \quad 1}{1} \quad \bar{2} \quad \frac{9 \ \bar{2}}{1} \quad 2$

$2 \quad 1 \quad ^{1} \quad 2 \quad 1$

1

$\frac{9 \ \overline{}}{}$,

(d) Substituting $\bar{2}$ into (3), we get $\bar{2}$ and 96. So, and are positive numbers. Hence, we know that points *A* and *B* are on the same branch. We may assume and . Then,

ob 1 d L ob 1 d LM ob 1 d LM

$\bar{9} \ LM \quad \bar{9} \ LM \quad \bar{9} \ \bar{2}$,

By direct calculation,

1

$\bar{2} \quad \bar{2} \quad \bar{2} \quad \bar{2} \quad 1 \ \bar{2} \quad \bar{2} \quad \bar{2} \quad \bar{2}$

$\bar{2} \quad 9 \quad \bar{2} \quad 2$

69 ,

The ob 1 d L is $9 \ \bar{3}$,

4. (a) (i) $\frac{\bar{0}7}{7} \quad \frac{\mathrm{lp}{-}7 \ \mathrm{pf} \ -}{\mathrm{lp}{-}7 \ \mathrm{pf} \ -} \quad \bar{9} \ \mathrm{lp} \frac{}{3} \frac{}{1} \ \mathrm{pf} \frac{}{3} \frac{}{1} \quad \bar{9} \ \mathrm{lp} \ — \ \mathrm{pf} \ —$

(ii) $\frac{\bar{0}7}{7}^{3} \quad \bar{9}^{3} \ \mathrm{lp} \ — \ \mathrm{pf} \ — \ ^{3}$

$9^{0} \ \mathrm{lp} \ \overline{\ 3\ } \ \mathrm{pf} \ \overline{\ 3\ }$

$9^{0} \ \mathrm{lp} \ \frac{}{3} \ \mathrm{pf} \ \frac{}{3}$

$9 \quad \bar{0} \quad 9$

(b) The results follow from the sum and difference of

$^{k} \ \mathrm{lp}k \ \mathrm{pf} \ k$ and $^{k} \ \mathrm{lp} \ k \ \mathrm{pf} \ k \ \mathrm{lp}k \ \mathrm{pf} \ k$.

$\mathrm{pf} \quad \mathrm{lp}^{0} \quad — \quad - \quad ^{0}$

$\frac{}{0} \quad 9 \quad ^{0} \ 0 \quad 0 \quad ^{0}$

$\frac{}{0} \quad ^{2} \quad ^{2} \quad ^{0} \quad ^{0} \quad 9$

$\frac{}{3} \ 9 \ \mathrm{lp} \quad \mathrm{lp}0 \quad \mathrm{lp}2$

(c)

9 l p l p0 l p2 3 pf l p0

pf l o l p

h l d $\frac{}{9}$ h h fp f bdbo

$\frac{k}{9}$ k fp f bdbo,

5. (a)

(b)(i) (E) has a unique solution if and only if h 2 h, that is, h 0.

(ii) When h 0, the system of equations becomes 0 2 0 . 0

Solving 0 2 0, we get 9 , .

This solution also satisfies the third equation and hence is the general solution of the system of equations.

(c) Suppose the system of equations has a solution. Then there exists such that

9 9 9 0 , that is, 0 , Hence, we know that the maximum value of is 3. When 0, we have and the solution of the system of equations is